Analytical Formulas for the Drain Current of Silicon Nanowire MOSFET

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Outline

- Background
- Compact model
- Derivation and characteristics of analytic models
- Summary
Background Ⅰ

- MOSFETs are downsized into nanometer scale. 3-Dim structures like FinFETs and Tri-gate MOS are actively investigated.

- Nanowire (NW) FETs attract wide attention for the excellent controllability against SCE.

- We have proposed a compact model of NW MOSFET. But complex “numerical analysis” is required in the drain current evaluation.
Analytical formulas are required for the circuit application of NW MOSFET. 

\[ I_D = \frac{W}{L} \mu C_{ox} (V_G - V_t - \frac{V_D}{2})V_D, \]
\[ I_D = \frac{1}{2} \frac{W}{L} \mu C_{ox} (V_G - V_t)^2 \]

As the analytical formula for nano-MOSFET, Lundstrom’s formula, e.g. 
\[ I_D = C_{eff} (V_G - V_t) \frac{1-R}{1+R} v_{\text{inj}} \]

is well-known, but only the saturation current is given, no linear region.

Objective
Derive the analytical drain-current models of NW MOSFET, manageable with “handy calculators”.
Starting point is the Compact Model

Essentials of Carrier scattering in Compact Model

Linear Potential Approx. : Electric Field $E$

$\varepsilon \sim k_B T$

Source

Channel

Initial Elastic Zone

Energy Relax Zone

Elastic Backscatt.

Elastic Backscatt. + (Optical Phonon Emission)

Transmission Probability : $T_i$

Optical Phonon

Electric Field $E$

To Drain

$F(0)$

$G(0)$

$V(x)$

$0$

$x_0$

$x$
Résumé of the Compact Model

\[ I_D = \frac{q}{\pi h} \sum_i g_i \int \left[ f(\varepsilon, \mu_S) - f(\varepsilon, \mu_D) \right] T_i(\varepsilon) d\varepsilon \]

\[ \mu_S - \mu_D = qV_D \]

\[ (V_G - V_i) - \frac{\mu_S - \mu_0}{q} = \frac{|Q|}{C_G}, \]

\[ \frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_{inv}}, \]

\[ C_{ox} = \frac{2\pi \varepsilon_{ox}}{\ln \left( \frac{r + t_{ox}}{r} \right)}. \]

(Electrostatics requirement)

\[ |Q| = \frac{q}{\pi} \sum_i g_i \left[ \int_{-\infty}^{+\infty} \frac{dk}{1 + \exp \left\{ \frac{\varepsilon_i(k) - \mu_S}{k_B T} \right\}} - \int_{-\infty}^{0} \frac{1}{1 + \exp \left\{ \frac{\varepsilon_i(k) - \mu_S}{k_B T} \right\}} - \int_{0}^{+\infty} \frac{1}{1 + \exp \left\{ \frac{\varepsilon_i(k) - \mu_D}{k_B T} \right\}} \right] T_i(\varepsilon_i(k)) dk \]

\[ T(\varepsilon) = \frac{\sqrt{2D_0}qE}{\left( \sqrt{B_0 + D_0} + \sqrt{D_0} \right)qE + \sqrt{2mD_0B_0} \ln \left( \frac{qEx_0 + \varepsilon}{\varepsilon} \right)} \]

(Carrier distribution in Subbands)

“Compact Model” still requires a complex calculation.
Is the “Compact Model” a reliable starting point?

Compact Model Result

Numerical simulation (Jin et al.)


[ NW Device parameters : \(D=5\) nm, \(L=15\) nm, \(t_{ox}=0.8\) nm, \(\mu=300\) cm\(^2\)/Vs ]

Agreement is satisfactory.
We trust the model and derive analytic formulas.
“Thick cylindrical NW” is assumed, promising in realistic application.

\[ \psi(r, \theta, x) = A \varphi_j(r) \exp(i n \theta) \exp(i k_x x) \]

\[ \varepsilon_{jn}(k_x) = \varepsilon_j + \frac{\hbar^2 n^2}{2 m r_0^2} + \frac{\hbar^2}{2m} k_x^2 \]

Subband summation is simplified.

“Average transmission coefficient” is sorted out.

\[ \int (f(\mu_s, \varepsilon) - f(\mu_D, \varepsilon)) T_i(\varepsilon) d\varepsilon \Rightarrow T_i(\varepsilon) \int (f(\mu_s, \varepsilon) - f(\mu_D, \varepsilon)) d\varepsilon \]

Energy integration is simplified.
Two approaches are possible.

[1] Full-degeneracy Model

- Temperature dependence is not clear in Ballistic characteristics.
- Actual temperature dependence is controlled by carrier scattering.
- Carrier scattering is treated separately.

We can assume Full-degeneracy in carrier distribution.

Fermi Distribution

\[
\frac{1}{1 + \exp\left(\frac{\epsilon - \mu}{k_B T}\right)}
\]

Step Distribution

\[\theta(\mu - \epsilon)\]

"Energy integration" is simplified.
### Full-degeneracy Model

**Drain current formula**

\[
\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_{inv}}, \quad C_q = q^2 \frac{m}{\pi \hbar^2} (2\pi r_0)
\]

\[
T = \frac{2qV_D}{\left(\sqrt{B_0 + D_0} + \sqrt{D_0}\right)qV_D + \sqrt{2mD_0B_0L \ln(3.15)}}
\]

\[
V_{D_{sat}} = \frac{C_G(V_G - V_t)}{C_G + 2C_q(2 - T)}, \quad \mu_S = \frac{C_G(V_G - V_t) + 2C_qTV_D}{C_G + 4C_q}
\]

\[
V_D \leq V_{D_{sat}}, \quad I_D = \frac{8}{3\pi q} \sqrt{\frac{2}{m} C_q T \left[ \mu_S^{3/2} - (\mu_S - qV_D)^{3/2} \right]}
\]

\[
V_{D_{sat}} \leq V_D, \quad I_D = \frac{8}{3\pi q} \sqrt{\frac{2}{m} C_q T \left(qV_{D_{sat}}\right)^{3/2}}
\]
Full-degeneracy Model

Device characteristics

Wire diameter: 10nm, \( L = 20 \text{nm} \) (SCE: not considered)
\( T_{ox} = 1 \text{nm}, \quad C_{inv}: \text{equivalent to } 1.5 \text{nm Si}, \quad \alpha = 1 \)
\( B_0 = 1.54 \times 10^{12} \text{eV}^{1/2} \text{s}^{-1} \) (equivalent to \( \mu = 300 \text{cm}^2/\text{Vs} \), \( D_0 = 1.46 \times 10^{12} \text{eV}^{1/2} \text{s}^{-1} \)

Compact Model

Full-degeneracy Model

Similar, but the “compact model” current is larger!
The analytic approximation might be too coarse in a thin device with the “10 nm diameter!”
Two approaches


When Fermi Temperature

\[ T_F \sim \frac{\mu_S - \mu_0}{k_B} < T \]

carrier distribution is Non-degenerate.

Actual distribution: Degeneracy \( \leftrightarrow \) Non-degeneracy

We can assume Boltzmann statistics as opposite to [1].

Fermi Distribution

\[ \frac{1}{1 + \exp\left(\frac{\varepsilon - \mu}{k_B T}\right)} \]

Boltzmann Distribution

\[ \exp\left[\frac{\mu - \varepsilon}{k_B T}\right] \]

“Energy integration” is analytically executed.
**Boltzmann Statistics Model**

**Drain current formula**

\[
\frac{1}{C_{\text{eff}}} = \frac{1}{C_{\text{ox}}} + \frac{1}{C_{\text{inv}}} + \frac{1}{C_{Q}},
\]

\[
v_{\text{inj}} = \sqrt{\frac{2k_B T}{\pi m}}
\]

\[
R = 1 - \frac{2qV_D}{\left(\sqrt{B_0 + D_0} + \sqrt{D_0}\right)qV_D + \sqrt{2mD_0 B_0 L} \ln(3.15)}
\]

\[
I_D = C_{\text{eff}} (V_G - V_t) v_{\text{inj}} \frac{(1 - R) \left(1 - \exp\left(-\frac{qV_D}{k_B T}\right)\right)}{\left(1 + R + (1 - R) \exp\left(-\frac{qV_D}{kT_B}\right)\right)}
\]
Boltzmann statistics Model

Device characteristics

Wire diameter: 10nm, \( L = 20\,\text{nm} \) (SCE: not considered)

\( T_{\text{ox}} = 1\,\text{nm}, \quad C_{\text{inv}} : \text{equivalent to}\,1.5\,\text{nm Si}, \quad \alpha = 1 \)

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Full-degeneracy Model

Boltzmann Statistics Model

Similar characteristics, Boltzmann Model slightly larger.
**Saturation Current expression**

(1) **Full-degeneracy Model**

\[ I_{D_{sat}} = C_{eff} (V_G - V_t) \frac{1-R}{1+R} v_{inj \ deg} \]

\[ v_{inj \ deg} = \frac{4h \sqrt{C_{eff} (V_G - V_t)}}{3m\pi \sqrt{2qr_0 (1+R)}} \]

Injection velocity of degenerate carriers

(2) **Boltzmann statistics Model**

\[ I_{D_{sat}} = C_{eff} (V_G - V_t) \frac{1-R}{1+R} v_{th} \]

\[ v_{th} = \sqrt{\frac{2k_B T}{\pi m_x}} \]

Thermal injection velocity

Both models are reduced to the **Lundstrom Formula** with an appropriate “injection velocity.”
Analytic drain current models of NW FETs, manageable with handy calculators, are derived.

The models meet the case of thick NW FETs, promising in realistic application. The models represent both limits of the full-degeneracy of carriers and the Boltzmann distribution.

The both cases provide a similar magnitude of drain current, which are smaller than the Compact Model result in a “10 nm diameter” device.

The saturation drain current is reduced to the Lundstrom formula in both cases.